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BAK3: Introduction to Quantitative Methods

Week 5: Univariate Descriptive Statistics II

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The Plan for today

- Statistics:
  - Measures of Position: **quantiles**.
  - Measures of Spread (or “Dispersion” or “Variability”): the **interquartile range**, the **variance** and the **standard deviation**.
- Coding in R:
  - Measures of Spread in R and Boxplots.

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Measures of Position

- By ‘position’ (or relative standing) we mean the **location of a data point within a variable by comparing it to other values.**
- We already know one measure of position: the **median**. It’s the mid-point of the variable: 50% of values are smaller, 50% are larger.
- Today: **quantiles**, as a generalisation of the mean.
- Other measure of position (later in the course, briefly): z-score.

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Quantiles

- The n quantile of a distribution is the point such that $(n \times 100)\%$ of the observations fall below that point.
- In this sense, the median is the **0.5 quantile**. But you can also get the 0.25, the 1/3, the 0.9 quantiles etc…
- Often described as **percentile**: The p^{th} percentile of a distribution is the point such that $p\%$ of observations fall below that point.

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Quantiles and Quartiles

- Key quantiles:
- 0.25 quantile, known also as ***first quartile***, or ***lower quartile***, or ***Q1*** or ***25th percentile***. 25% (one-quarter) of observations fall below that point; 75% (three-quarters) of observations fall above.
- 0.75 quantile, known also as ***third quartile***, or ***upper quartile*** or ***Q3*** or ***75th percentile***. 75% (three-quarter) of observations fall below that point; 25% (one-quarter) of observations fall above.

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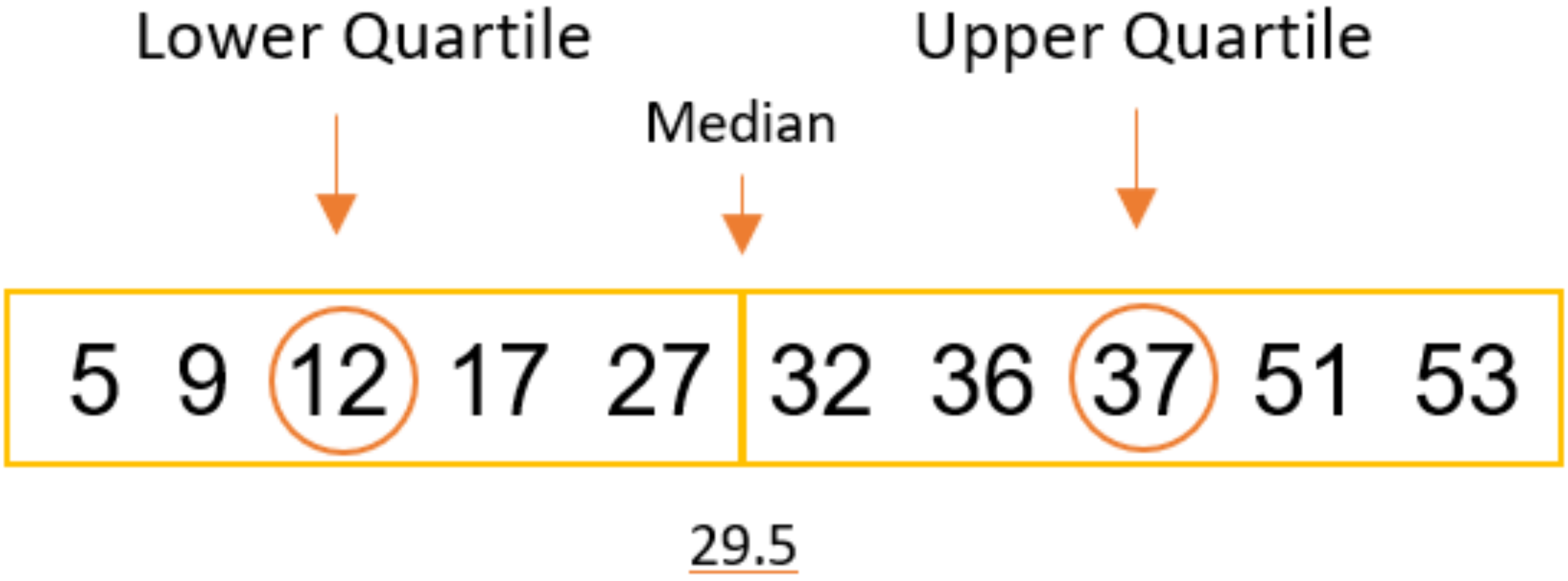
Quartiles



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Interquartile Range

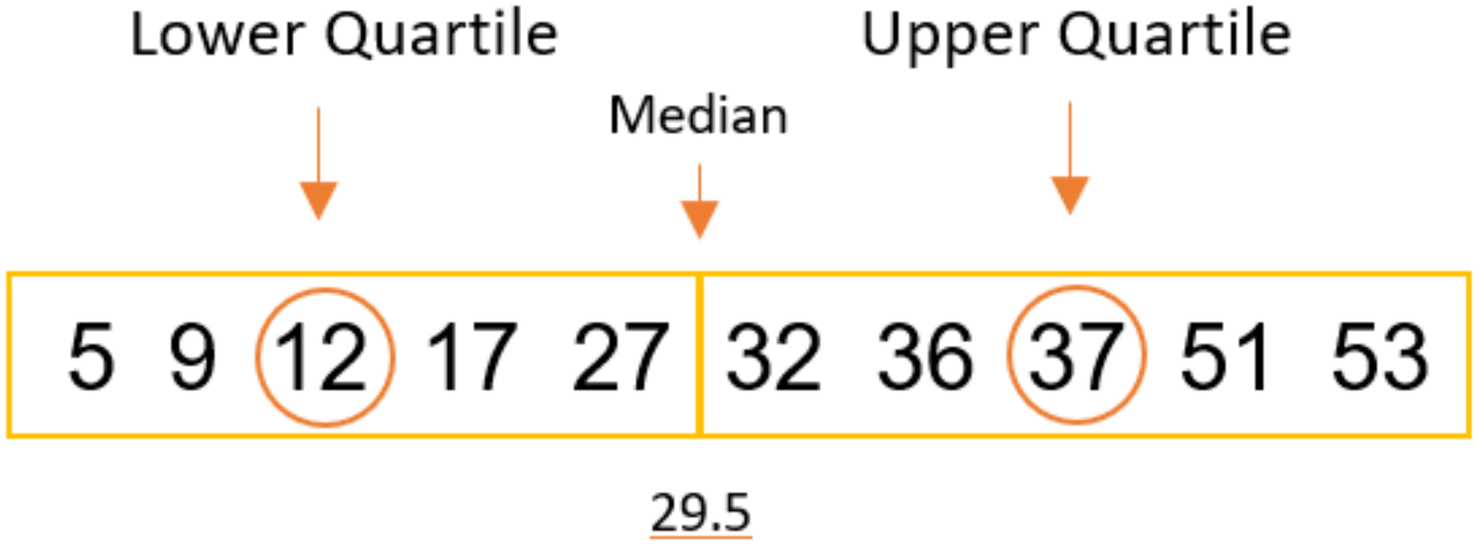
- We like the first and third quartile (Q1 and Q3) because 50% of values fall between these two, giving us a sense of how far from the median we find half of the population distribution.
- So, our first measure of spread is the **interquartile range**:

$$IQR = Q3 - Q1$$

  - The ‘wider’ the IQR, the more variability we have in our variable.

Interquartile Range

- Q3 = 37
- Q1 = 12
- **IQR** = 25



- Why should we care?
  - Definition of ***outliers*** is done with reference to the IQR.
  - Used in a common form of data visualisation: the ***boxplot***.

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Outliers

- An observation is an ***outlier*** if it falls more than 1.5(IQR) above the upper quartile (Q3) or more than 1.5(IQR) below the lower quartile (Q1).
- For example: wages = {1800€, 2035€, 2300€, 2400€, 3200€, 3550€, 3900€, 5500€, 7000€, 500000€}, where **Q1**=2300€ and **Q3**=5500€.
  - **IQR** = 5500-2300 = 3200
- The observation with a wage of 500000€ is an outlier, because…
  - Q3 + 1.5(IQR) = 5500 + 1.5(3200) = 10300; and 500000 > 10300

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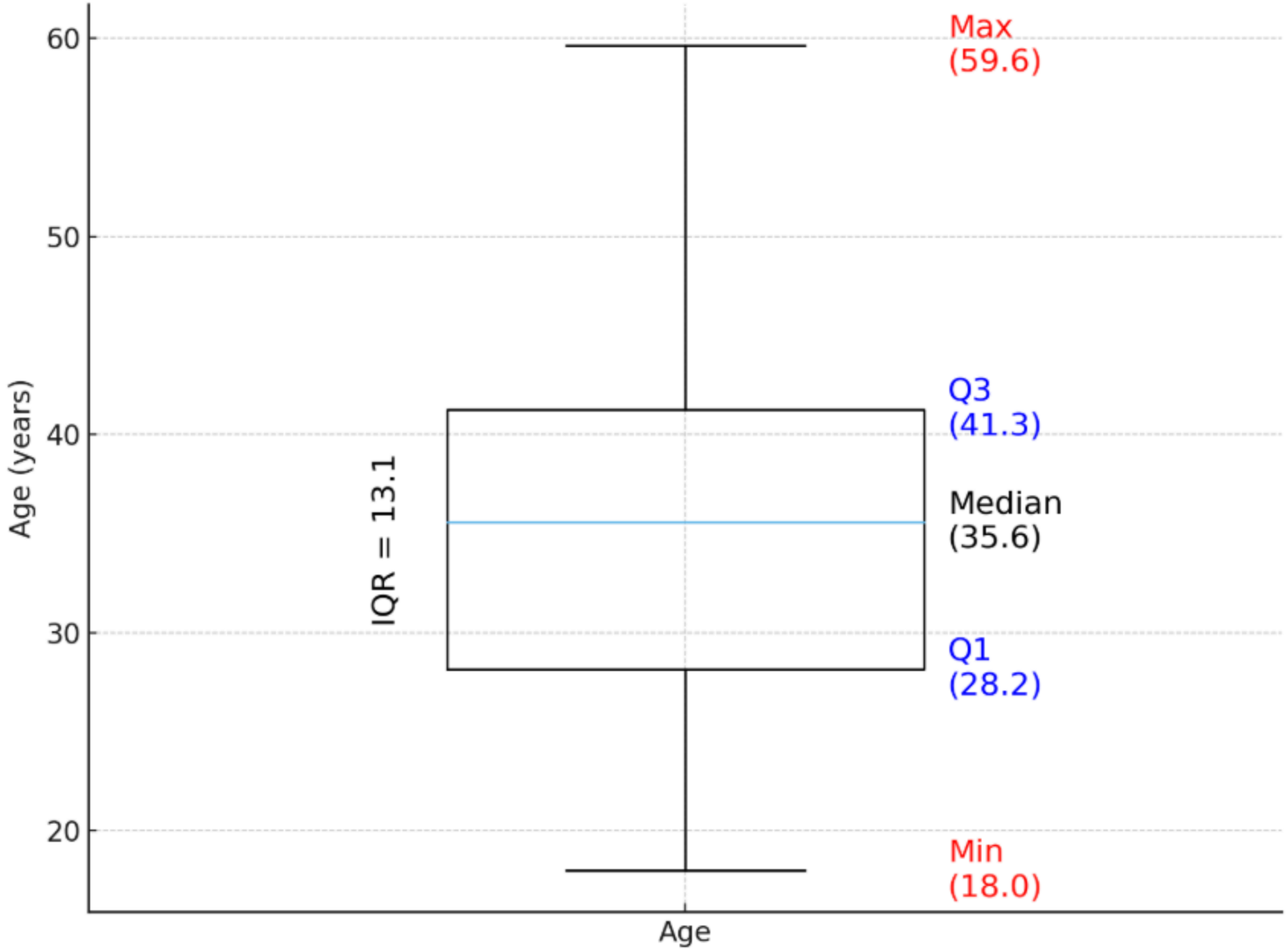
Boxplots



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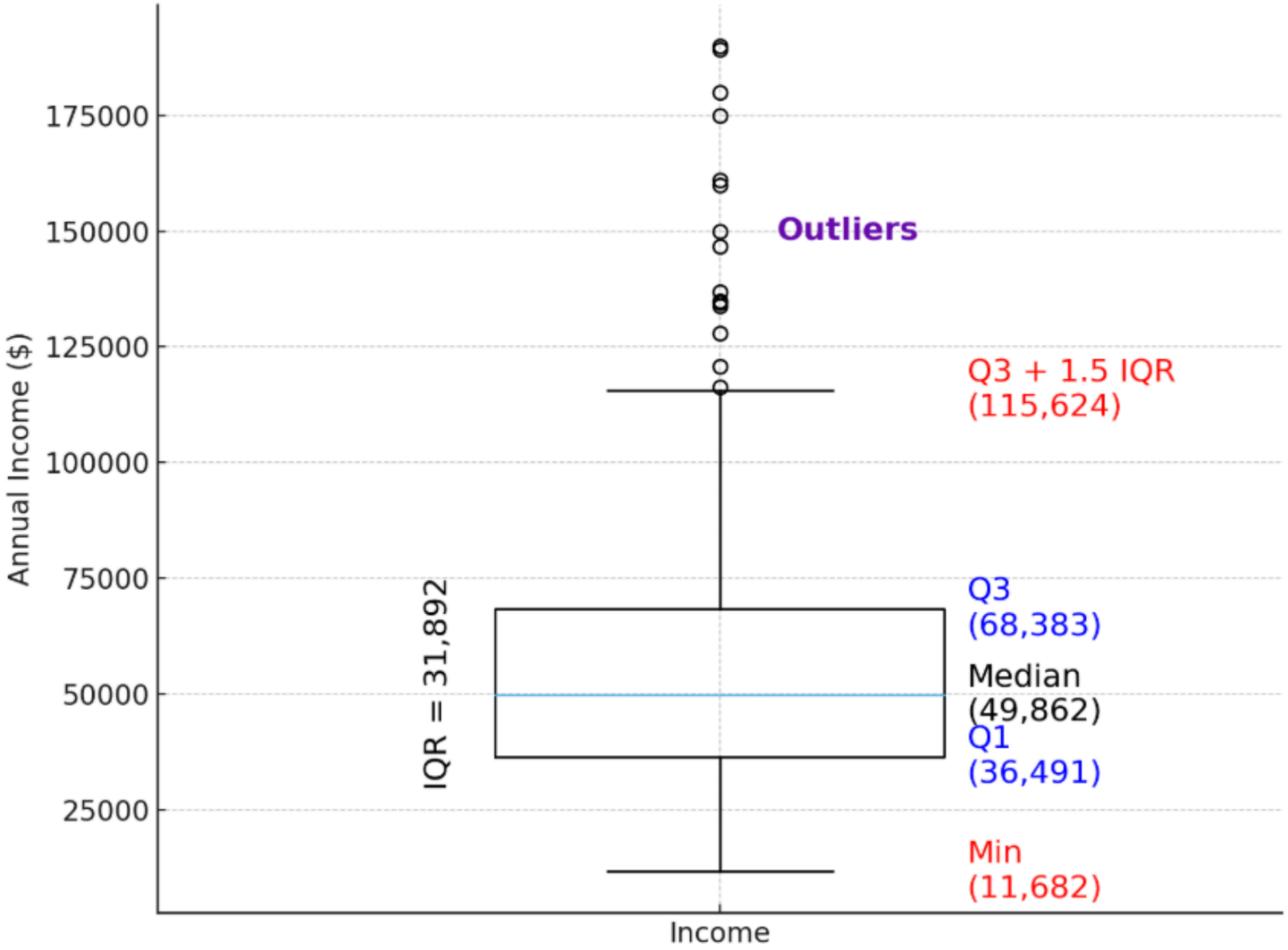
Boxplots (with outliers)



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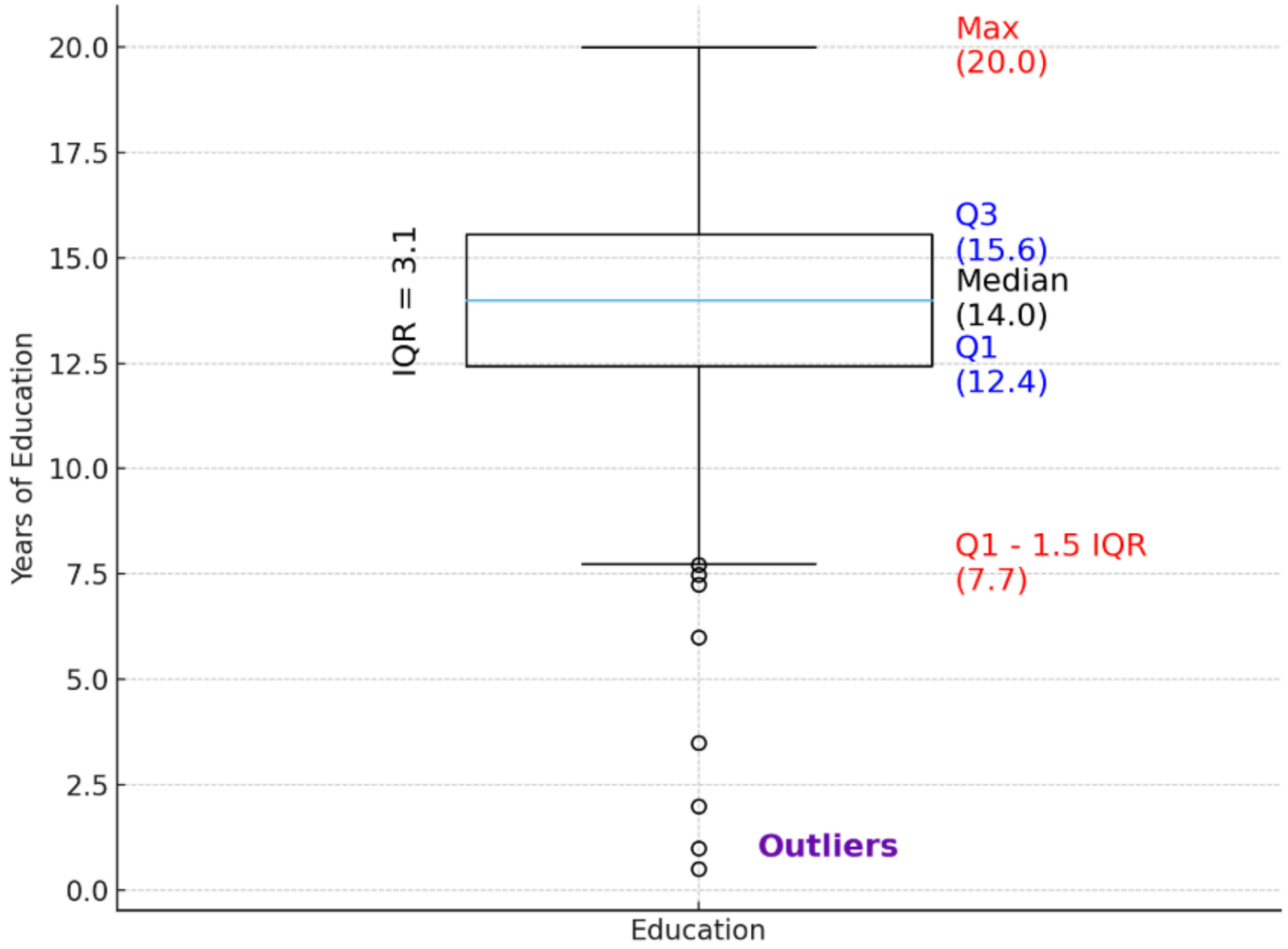
Boxplots (with outliers)



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Variance

- The variance is another measure of spread that assesses how different each observation is from the **mean** observation.
- The formula is:

$$\mathrm{Var}(X) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

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Variance

$$\mathrm{Var}(X) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- Breaking it down:
  - $(x_i - \bar{x})$ means: take the **differences between every single value** $x_1, x_2, x_3 \ldots x_n$ of our variable X, **and the mean** of X, which is $\bar{x}$.
  - Then, we **square all** of them, so all the signs of these ‘deviations’ become positive.
  - The summation sign Σ means: **add up** all these squared deviations.
  - Then, divide by the number of observations **minus one**: $n - 1$.

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Variance of $X = \{13,16,8,5,8,7\}$

- Step 1: find the mean $\bar{x} = (13 + 16 + 8 + 5 + 8 + 7)/6 = 9.5$
- Step 2: $(x_i - \bar{x}) = \{13 - 9.5; 16 - 9.5; 8 - 9.5; 5 - 9.5; 8 - 9.5; 7 - 9.5\}$
- Step 3: $(x_i - \bar{x})^2 = \{3.5^2; 6.5^2, -1.5^2, -4.5^2; -1.5^2; -2.5^2\}$
- Step 4: $\sum_{i=1}^{n} (x_i - \bar{x})^2 = 12.25 + 42.25 + 2.25 + 20.25 + 2.25 + 6.25 = 85.5$
- Step 5: divide by $n - 1$. Which is to say $85.5/(6 - 1) = 17.1.$

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Why does this work?

- The variance is a measure of spread because it gets smaller as values ‘cluster’ around the mean, and bigger as observations ‘spread out’.
- Consider these two variables with the **same mean:**
  - ***No variation.*** $X = \{3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3...\}$ with $\bar{x} = 3$.
  - ***Lots of variation.*** $X = \{+2250, -815, -1426...\}$ with $\bar{x} = 3$.

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Why does this work?

- ***No variation***. $X = \{3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3...\}$ then $(x_i - \bar{x}) = \{0,\ 0,\ 0,\ 0,\ 0...\}$, then $\mathrm{Var}(X) = 0.$
- ***Lots of variation.*** $X = \{+2250, -856, -1456...\}$, then $(x_i - \bar{x}) = \{\text{Big Number}, \ -\text{Big Number}, \ -\text{Big Number}\ldots\}$, then $(x_i - \bar{x})^2 = \{\text{Huge Number}, \text{Huge Number}, \text{Huge Number}\ldots\}$. Sum them all up and divide by $n - 1$, $\mathrm{Var}(X) =$ **Big Positive Number.**

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Unsatisfactory Answers to Very Good Questions

- **Why do we take the squares and not the absolute value of deviations?** Mostly for mathematical convenience: squaring has nice algebraic properties that the absolute value does not. Also connected with the ‘least squares’ property of the mean from last week.
- **Why do we divide by $n-1$ and not by n?** You could, and some formulas for the variance do divide by n. The ‘minus one’ is known as **Bessel’s correction**. We divide by $n-1$ when we want to compute the sample variance, we divide by n when we want the population variance. Here, we mostly work with samples, so usually we want $n-1$. But don’t worry about this now.

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The Standard Deviation

- The standard deviation is simply the **square root of the variance:**

$$s_X = \sqrt{\mathrm{Var}(X)} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- We normally prefer to discuss the standard deviation because it's in the same unit of measurement (cm, percentage points, number of children, euros, years etc.) as the original variable.

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The Standard Deviation

- For instance, if our variable is people's years = {59, 23, 42, 31, 18, 38}
- The variance is 216.6 "squared years". No immediate sense.
- The std. deviation is 14.7 **years**. For instance, we could say that the 23 year-old is about one standard deviation younger than the mean.

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The Standard Deviation

- **Properties of the standard deviation:**
  - It is always positive (at most it can be zero).
  - The greater the variability around the mean, the larger it gets.
  - Like the mean (and unlike the IQR and the median), it is affected by all values of the variable, **including outliers**.
- ***Notation***: normally, the standard deviation is notated as σ or as s. The variance is notated in terms of the standard deviation as σ^2 or as s^2.

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Summing Up…

- **Q1** and **Q3** bracket 50% of observations in the middle of a variable distribution; they are used to compute the **interquartile range**.
- **Boxplot**: visual summary of key points in a variable's distribution: minimum*, Q1, median, Q3, maximum*; outliers if there are any.
- The **standard deviation** is the square root of the **variance**. Both measures summarise the spread of the variable in a single positive number, but the former is more common and easier to interpret.
- And now, let's open RStudio…